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Performance of annular fins with different profiles subject to variable heat transfer coefficient

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Abstract

Performance of annular fins of different profiles subject to locally variable heat transfer coefficient is investigated in this paper. The performance of the fin expressed in terms of fin efficiency as a function of the ambient and fin geometry parameters has been presented in the literature in the form of curves known as the fin-efficiency curves for different types of fins. These curves, that are essential in any heat transfer textbook, have been obtained based on constant convection heat transfer coefficient. However, for cases in which the heat transfer from the fin is dominated by natural convection, the analysis of fin performance based on locally variable heat transfer coefficient would be of primer importance. The local heat transfer coefficient as a function of the local temperature has been obtained using the available correlations of natural convection for plates. Results have been obtained and presented in a series of fin-efficiency curves for annular fins of rectangular, constant heat flow area, triangular, concave parabolic and convex parabolic profiles for a wide range of radius ratios and the dimensionless parameter m based on the locally variable heat transfer coefficient. The deviation between the fin efficiency calculated based on constant heat transfer coefficient, reported in the literature, and that presently calculated based on variable heat transfer coefficient, has been estimated and presented for all fin profiles with different radius ratios.

1. Introduction

Annular fins find numerous applications in compact heat exchangers, in specialized installations of single- and double-pipe heat exchangers, in electrical apparatus in which generated heat must be efficiently dissipated, on cylinders of air cooled internal-combustion engines, etc. In a conventional heat exchanger heat is transferred from one fluid to another through a metallic wall. The rate of heat transfer is directly proportional to the extent of the wall surface, the heat transfer coefficient and to the temperature difference between one fluid and the adjacent surface. If thin strips (fins) of metals are attached to the basic surface, extending into one fluid, the total surface for heat transfer is thereby increased. It might be expected that the rate of heat transfer per unit of the base surface area would increase in direct proportion. However, the average surface temperature of these strips (fins), by virtue of temperature gradient through them, tends to decrease approaching the temperature of the surrounding fluid so the effective temperature difference is decreased and the net increase of heat transfer would not be in direct proportion to the increase of the surface area and may be considerably less than that would be anticipated on the basis of the increase of surface area alone. The use of fins in one side of a wall separating two heat-exchanging fluids is exploited most if the fins are attached to or made an integral part of that face on which the thermal resistivity is greatest. In such a case the fins serve the purpose of artificially increasing the surface transmittance.

The ratio of the actual heat transfer from the fin surface to that, that would transfer if the whole fin surface were at the same temperature as the base is commonly called as the fin efficiency. Harper and Brown [1], in connection with air-cooled aircraft engines, investigated straight fins of constant thickness, wedge-shaped

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Nomenclature

A	cross-sectional area of the fin perpendicular to the heat flow
g	gravitational acceleration (m/s^2)
h	ambient convection heat transfer coefficient (W/(m^2 K))
i	numerical index in radial direction
k_s	thermal conductivity of the solid (fin) (W/(m K))
k_f	thermal conductivity of the fluid (W/(m K))
L	length of the fin, $r_o - r_b$ (m)
m	dimensionless parameters, $L\sqrt{(h_u + h_l)/ky_b}$
Nu	Nusselt number, hr/k_f
N	index of the fin profile
P	perimeter of the fin cross-section perpendicular to the heat flow
Ra	Rayleigh number
R	dimensionless local radius, r/L
r	local radius of the fin

Greek symbols

α	thermal diffusivity (m^2/s)
β	volumetric coefficient of thermal expansion (K^{-1})
θ	dimensionless temperature, $\theta = (T - T_f)/(T_b - T_f)$
ν	kinematic viscosity of the ambient fluid (m^2/s)

Subscripts

b	condition or variable specified at the base of the fin
f	ambient fluid property
r	local value of the variable at r
o	condition or variable specified at the edge of the fin
s	solid, fin material property
u	for upper surface
l	for lower surface

straight fins and annular fins of constant thickness; equations for the fin efficiency of each type were presented and the errors involved in certain of the assumptions were evaluated.

Schmidt [2] studied the same three types of fins from the material economy point of view. He stated that the least metal is required for given conditions if the temperature gradient is linear, and showed how the thickness of each type of fin must be varied to produce this result. Finding, in general, that the calculated profiles were impractical to manufacture, Schmidt proceeded to show the optimum dimensions for straight and annular fins of constant thickness and for wedge-shaped straight fins under given operating conditions. The temperature gradient in conical and cylindrical spines was determined by Focke [3]. In this work, Focke, like Schmidt, showed how the spine thickness must be varied in order to keep the material requirement to a minimum; he, too, found that the result is impractical and went to determine the optimum cylindrical- and conical-spine dimensions.

Murray [4] presented equations for the temperature gradient and the effectiveness of annular fins with constant thickness with a symmetrical temperature distribution around the base of the fin. Carrier and Anderson [5] discussed straight fins of constant thickness, annular fins of constant thickness and annular fins of constant cross-sectional area, presenting equations for fin efficiency of each. In the latter two cases the solutions were given in the form of infinite series.

Avrami and Little [6] derived equations for the temperature gradient in thick-bar fins and showed under what conditions fins might act as insulators on the basic surface. Approximate equations were also given including, as a special case, that of Harper and Brown [1]. A rather unusual application of Harper and Brown's equation was made by Gardner [7], in considering the ligaments between holes in heat-exchanger tube sheets as fins and thereby estimating the temperature distribution in tube sheets.

Gardner [8] derived general equations for the temperature gradient and fin efficiency in any extended surface to which a set of idealized assumptions are applicable. In this regard, Gardner [8] presented analytical solutions for fin efficiency for straight fins and spines with different profiles and annular fins of rectangular and constant heat flow area profiles subject to constant heat transfer coefficient. Ullmann and Kalman [9] extended the work of Gardner [8] concerning the annular fins and presented the fin efficiency along with the optimized dimensions for annular fins with different profiles. The effect of fin parameters on the radiation and free convection heat transfer from a finned horizontal cylindrical heater has been studied experimentally by Karaback [10]. The fins used were circular fins. The experimental setup was capable of analyzing the effect of fin diameter and spacing on heat transfer. A correlation equation for the tip temperature of uniform annular fins as a function of thermogeometric parameters and radii ratio has been obtained by Campo and Harrison [11]. In this study, Campo and Harrison considered constant heat transfer coefficient along the fin. The optimum dimensions of circular fins of trapezoidal profile with variable thermal conductivity and heat transfer coefficients have been obtained by Razelos and Imre [12]. In this work, Razelos and Imre considered the linear variation of the thermal conductivity with temperature and assumed that the heat transfer coefficients vary according to a power law with distance from the bore. Performance and optimum dimensions of longitudinal and annular fins and spines with a temperature-dependent heat transfer coefficient have been presented by Laor and Kalman [13]. In this work, Laor and Kalman considered the heat transfer coefficient as a power function of temperature and used exponent values in the power function that represent different heat transfer mechanisms such as free convection, fully developed boiling and radiation. The optimum dimensions of circular fins with variable profile and temperature-dependent thermal conductivity have been obtained by Zubair et al. [14]. A correlation for the optimal dimensions of a constant and variable profile fins was presented in terms of a reduced heat transfer rate. Assuming that the heat transfer coefficient is a power function of the temperature difference of a straight fin of a rectangular profile and that of the ambient, Unal [15] obtained a closed form solution for the one-dimensional temperature distribution for different values of the exponent in the power function. An exact solution for the rate of heat transfer from a rectangular fin governed by a power law-type temperature dependence heat transfer coefficient has been obtained by Sen and Trinh [16]. With the help of symbolic computational mathematics, Campo and Stuffle [17] presented a simple and compact form correlation that facilitates a rapid determination of fin efficiency and tip temperature in terms of fin controlling parameters for annular fins of constant thickness. Lien-Tsaiyu and Chen [18] presented the transient temperature response of a convective–radiative rectangular profile annular fin under a step temperature change occurring in its base. They have assumed constant heat transfer coefficient along the fin and used a hybrid method of Taylor transformation and finite difference approximation. The temperature distribution was implemented by employing natural cubic spline fitting.

From the thorough literature survey summarized above, the author found that there is no work in the literature that reported the effect of temperature-dependent heat transfer coefficient on the fin efficiency of annular fins with different profiles subject to natural convection except the work presented by Laor and Kalman [13]. No attention has been given in the literature to the effects of local variations of the heat transfer coefficient on the upper and lower surfaces of horizontal annular fins with different profiles subject to natural convection. The aim of the present article is to investigate such effects. This type of study would be of direct use by the heat transfer equipment designers and rating engineers.

2. Mathematical model and assumptions

The mathematical analysis, in the above cited articles, for the heat transfer from fins, was based on some or all of the following assumptions:

1. Steady heat flow.
2. The fin material is homogeneous and isotropic.
3. There are no heat sources in the fin itself.
4. The heat flow to or from the fin surface at any point is directly proportional to the temperature difference between the surface at that point and the surrounding fluid.
5. The thermal conductivity of the fin is constant.
6. The heat transfer coefficient is the same over all the fin surface.
7. The temperature of the surrounding fluid is constant.
8. The temperature of the base of the fin is uniform.
9. The fin thickness is so small compared to its length and width that temperature gradient normal to the surface may be neglected.
10. The heat transferred through the outermost edge of the fin is negligible compared to that passing through the sides.

Of these assumptions, only 5, 6, 8, 9, and 10 are open to serious question. The error involved in assumptions 9 and 10 has been investigated by Harper and Brown [1], and Avrami and Little [6], for straight fins of constant thickness. These investigations showed that the error due to these two assumptions (9 and 10) is very small for most practical forms of extended surfaces. The question of temperature variation at the base of the fin might be important for radial fins and is not apt to arise for other types [6]. Moreover, for symmetrical radial fins this question may not arise as well. The effect of temperature-dependent thermal conductivity on the performance of annular fins with different profiles has been addressed by Razelos and Imre [12] and Zubair et al. [14].

On the other hand, with some situations, the heat transfer coefficient undoubtedly does vary from point to point on the fin [12–16], specially if the natural convection is the dominant mode of heat transfer in the fluid surrounding the fin. The main objective of this paper is to study the effect of the local heat transfer coefficient along the upper and lower surfaces of a fin on the fin performance represented by the fin efficiency for annular fins of different profiles subject to natural convection.

Fig. 1(a) depicts a general annular fin profile and shows the main geometric profile parameters. The fin profile is defined according to the variation of the fin thickness along its extended length. The general equation of the radial fin profiles studied in the present article is

$$y_r = y_b(R_o - R)^n,$$

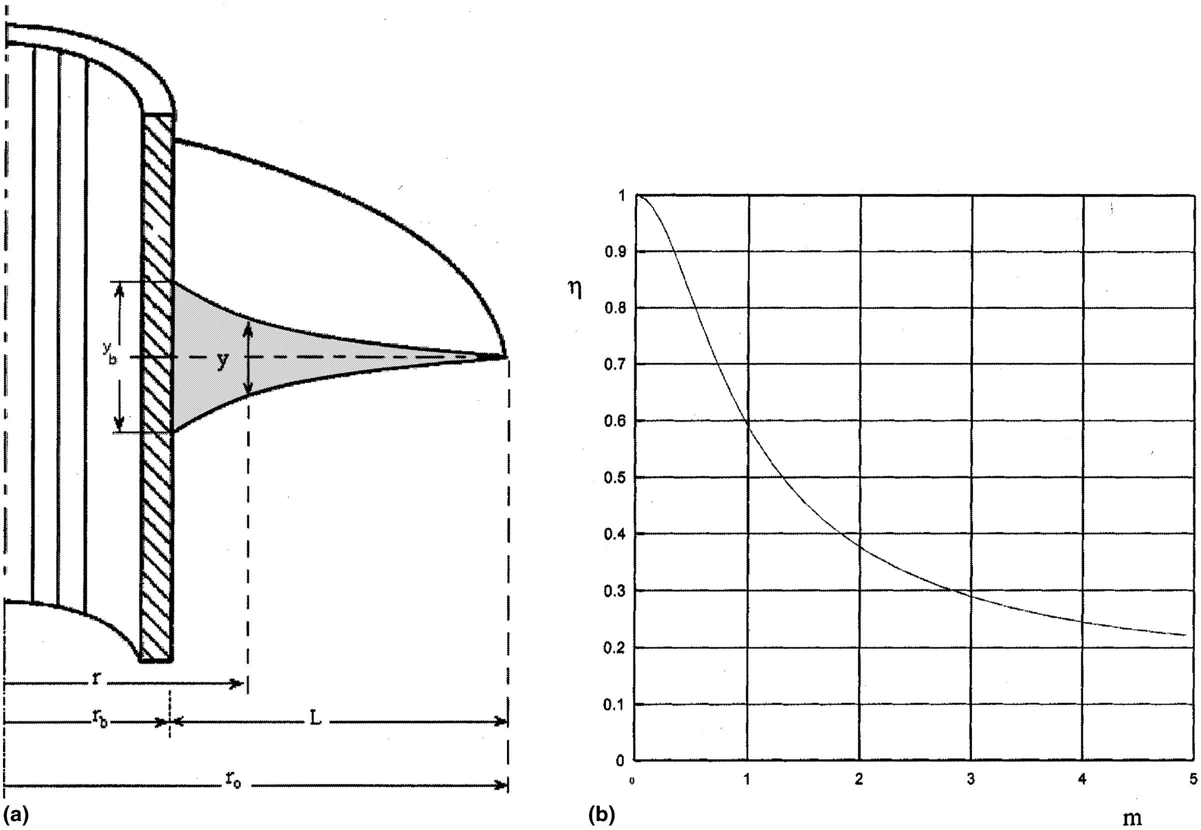

Fig. 1. (a) General annular fin profile. (b) Comparison of the solution based on Eq. (3), – – for $ds \neq dr$ and solution based on Eq. (4), ------ for $ds = dr$.

where n is the profile index; $n = 0$ represents the constant thickness fin which has a rectangular profile. $n = 1/2$ corresponds to the convex parabolic fin profile while $n = 1$ describes the triangular fin profile with straight surfaces. The value of $n = 2$ gives the concave parabolic profile. All the fin profiles considered in the present study start with a thickness y_b at the base. The triangular, convex parabolic and concave parabolic profiles have tips at their ends (i.e, $y = 0$ at $r = r_o$) while the rectangular has a constant thickness along the fin. The annular fins with constant area for heat flow have a hyperbolic profile. For such a profile, the thickness of the fin varies with the radius such that $y \cdot r = \text{constant}$, and the profile can be expressed as

$$y_r = y_b\left(\frac{R_b}{R}\right)$$

the hyperbolic fin has a sharp edge at infinity, but in practice, it is cut off at a distance r_o from the axis of symmetry. The general partial differential equation governing the steady heat transfer from all fins can be written as

$$\frac{d}{dr}\left(k_s A_r \frac{dT}{dr}\right) dr - A_s (h_u + h_l)_r (T - T_\infty) = 0,$$

where k_s is the fin material thermal conductivity, $A_r = 2\pi r y_r$ is the cross-sectional area perpendicular to the heat flow, and A_s is the local surface area at that section, $A_s = 2\pi r\, ds$ for annular fins. The above equation can be written for an annular fin in polar coordinates as

$$\frac{d}{dr}\left(k_s (2\pi r y_r) \frac{dT}{dr}\right) dr - 2\pi r\, ds (h_u + h_l)_r (T - T_\infty) = 0,$$

where ds is the arc length and the subscripts u and l mean the upper and lower surfaces for the case of horizontal fins and the subscript r means the local value at radius r. This equation can be rewritten as

$$\frac{d}{dr}\left(r y_r \frac{dT}{dr}\right) - r\frac{ds}{dr}\left(\frac{(h_u + h_l)_r}{k_s}\right)(T - T_\infty) = 0,$$

which can be rearranged and written as

$$y_r \frac{d^2 T}{dr^2} + \left(\frac{y_r}{r} + \frac{dy_r}{dr}\right)\frac{dT}{dr} - \left(\frac{(h_u + h_l)_r}{k_s}\right)\frac{ds}{dr}(T - T_\infty) = 0.$$

Dividing both sides by y_r, one can write:

$$\frac{d^2T}{dr^2} + \left(\frac{1}{r} + \frac{1}{y_r}\frac{dy_r}{dr}\right)\frac{dT}{dr} - \left(\frac{(h_u + h_l)_r}{k_s y_r}\right)\frac{ds}{dr}(T - T_\infty) = 0. \quad (1)$$

The local heat transfer coefficient, h_r, in the above equation should be the actual local heat transfer coefficient which could be obtained from experimental measurements or correlations that give the actual local heat transfer coefficient for free convection from non-isothermal horizontal plates. According to the author information, such experimental data or correlations are not available in the literature. Due to the lack of such information and because of solving the above-mentioned equation, Eq. (1), numerically using finite difference approximation technique, the local heat transfer coefficient, h_r, will be calculated approximately using correlations that give the average Nusselt number for free convection from isothermal horizontal surfaces. This would be a good approximation in which the finite strip of the fin for which the governing equation is applied is considered locally isothermal. This approximated local heat transfer coefficient, h_r, will be calculated from the following equation:

$$h_r = \frac{Nu_r k_f}{r},$$

where k_f is the ambient fluid thermal conductivity, r is the local characteristic length (local radius of the fin) and Nu_r is the local Nusselt number which can be calculated based on the empirical natural convection correlations for plates [19]

Upper surface $Nu_r = 0.54\ Ra_r^{1/4}, \quad 10^4 \leqslant Ra \leqslant 10^7,$

Lower surface $Nu_r = 0.27\ Ra_r^{1/4}, \quad 10^5 \leqslant Ra \leqslant 10^{10},$

where

$$Ra_r = \frac{g\beta\theta r^3}{\nu\alpha}.$$

The governing equation, Eq. (1), can be rewritten for a general profile with index n in a dimensionless form as follows:

$$\frac{d^2\theta}{dR^2} + \left(\frac{1}{R} - \frac{n}{(R_o - R)}\right)\frac{d\theta}{dR} - \left(\frac{m^2}{(R_o - R)^n}\right)\frac{ds}{dr}\theta = 0, \quad (2)$$

where

$$m = L\sqrt{\frac{(h_u + h_l)}{k_s y_b}}.$$

The arc length, ds, in the above equation can be calculated approximately from the following equation:

$$ds = \left(\left(\frac{dy_r}{2}\right)^2 + dr^2\right)^{1/2} = dr\left(\left(\frac{dy_r}{2dr}\right)^2 + 1\right)^{1/2}$$

hence

$$\frac{ds}{dr} = \left(\left(\frac{dy_r}{2dr}\right)^2 + 1\right)^{1/2}.$$

Substituting the above relation, for the previously given general profile with index n, in Eq. (2), one can write:

$$\frac{d^2\theta}{dR^2} + \left(\frac{1}{R} - \frac{n}{(R_o - R)}\right)\frac{d\theta}{dR} - \left(\frac{m^2}{(R_o - R)^n}\right)\left(\left(\frac{-ny_b}{2L}(R_o - R)^{n-1}\right)^2 + 1\right)^{1/2}\theta = 0. \quad (3)$$

It is worth mentioning here that the incremental arc length ds on the arbitrary surface profile of the fin surface can be approximated by the incremental length in the radial direction, dr. This would introduce a numerical error in the solution. This error would be reduced if the slope of the fin surface profile is small (i.e., when the fin thickness is small which is usually the case for fins). Moreover, this error would be reduced for numerical solutions for small increments (i.e., by using very small mesh size in the numerical solution). The effect of this approximation on the accuracy of the solution will be presented in the Section 3 hereafter. If the approximation discussed above (i.e., ds = dr) is used, Eq. (2) can be simply written as

$$\frac{d^2\theta}{dR^2} + \left(\frac{1}{R} - \frac{n}{(R_o - R)}\right)\frac{d\theta}{dR} - \left(\frac{m^2}{(R_o - R)^n}\right)\theta = 0. \quad (4)$$

For annular fin with hyperbolic profile the governing equation will be:

$$\frac{d^2\theta}{dR^2} + \left(\frac{1}{R} - \frac{R_b}{R^2}\right)\frac{d\theta}{dR} - \left(\frac{m^2}{R_b/R}\right)\theta = 0. \quad (5)$$

These Eqs. (4) and (5), will be solved for thermal boundary conditions of having the base kept at constant and uniform temperature and the fin tip is kept thermally insulated. The above non-linear ordinary differential equations have been converted to algebraic equations using the finite difference techniques. The final finite difference form of Eqs. (4) and (5) can be written as follows:

$$\theta_i = \frac{\left(\frac{\theta_{i+1}+\theta_{i-1}}{\Delta R^2}\right) + \left(\frac{1}{R_i} - \frac{n}{(R_0 - R_i)}\right)\left(\frac{\theta_{i+1}-\theta_{i-1}}{2\Delta R}\right)}{\left(\frac{2}{\Delta R^2} - \frac{m^2}{(R_o - R_i)^n}\right)} \quad (6)$$

and for a profile with constant area of heat flow, this equation will be:

$$\theta_i = \frac{\left(\frac{\theta_{i+1}+\theta_{i-1}}{\Delta R^2}\right) + \left(\frac{1}{R_i} - \frac{R_b}{R_i^2}\right)\left(\frac{\theta_{i+1}-\theta_{i-1}}{2\Delta R}\right)}{\left(\frac{2}{\Delta R^2} - \frac{m^2}{(R_b/R_i)}\right)}. \tag{7}$$

Subject to boundary conditions: at $R = R_b$, $\theta = 1.0$ and at $R = R_o$, $d\theta/dR = 0$.

3. Results and discussions

The dimensionless forms of the governing equations (4) for variable profile fins and (5) for the constant area profile fin include two dimensionless controlling parameters in addition to the index n which defines the fin profile. These two parameters are namely the dimensionless variable m and the fin radius ratio which is implicitly inherited in evaluating the value of R_o in Eq. (4), $R_o = r_o/(r_o - r_b) = (r_o/r_b)/(r_o/r_b - 1)$ and the value of R_b in Eq. (5), $R_b = r_b/(r_o - r_b) = 1/(r_o/r_b - 1)$, where r_o and r_b are the outer and base radii of the fin, respectively. So, the fin performance will be expressed in the form of curves that give the fin efficiency as a function of these two dimensionless controlling parameters m and r_o/r_b.

The finite difference equations presented have been tested for the effect of mesh size on the accuracy of the solution. The numerical solution for an annular fin with concave parabolic profile and radius ratio 2 has been obtained via numerical meshes of 5, 10, 15 and 20 grid points. The numerical solution for this case showed independence on the grid size for mesh with grid points of 15 and above. The difference between the fin efficiency that is obtained numerically via a grid of 15 points with respect to that obtained via a grid of 20 points was 0.012%. So, a grid of 15 points has been adopted through out the work.

The effect of approximating the incremental arc length to the incremental length on the radial direction has been investigated for the concave fin profile since it has a surface profile of a large slope. The solution has been obtained for a concave fin with radius ratio 2 using Eq. (3) that takes the incremental arc length on the solution and Eq. (4) that takes the approximated radial increment instead of the incremental arc length. The comparison of the two solutions is presented in Fig. 1(b) for the above particular case. Fig. 1(b) shows that this approximation ($ds = dr$) has almost no effect on the accuracy of the solution. So, Eq. (4) has been used to get the solutions for all cases considered and Eq. (5) has been used to obtain the solution for the annular fin with constant area of heat flow. Moreover, the present numerical scheme, the solution algorithm and the solution computer code have been first bench marked via providing the numerical solution for simple cases that have readily available closed form analytical solution. These cases are namely; annular fins with rectangular profile and annular fins with constant heat flow area profile subject to constant heat transfer coefficient along the fin surface. The numerical solution and the analytical solution for the aforesaid cases were almost typical. Such a comparison was a validation for the finite difference scheme, the solution algorithm and the computer code used during the present study. Then, the program has been used to solve the heat transfer governing equations for the five considered types of the annular fin profiles subject to variable heat transfer coefficient that varies as a function of the local temperature along the fin surface. The program is used to solve the finite difference equations for all cases under study to get the temperature distribution along the fin. To solve these equations, one needs to evaluate the local values of the dimensionless parameter m which is a function of the local heat transfer coefficient which in turn is a function of the local temperature. Hence, the solution had to be of iterative nature. So, a special computer code has been designed and developed to solve this set of equations using Gauss–Seidel iterative method to obtain the local temperature distribution along the fin. This temperature distribution is then used to calculate the local heat transfer coefficient and then the actual local heat transfer rate along the fin. This actual local heat transfer rate is numerically integrated to calculate the overall actual heat transfer rate through the whole fin surface. The maximum possible heat transfer rate is also calculated locally based on the local heat transfer coefficient while the temperature was considered as if it were constant as that of the base. This local maximum possible heat transfer rate is integrated numerically to calculate the total maximum possible heat transfer rate through the fin. The ratio of the total actual heat transfer rate to the total maximum possible heat transfer rate was used during the present study as the fin efficiency, as used by Gardner [8], Ullmann and Kalman [9] and all heat transfer textbooks. The fin efficiency is then plotted against the dimensionless parameter m that is calculated locally and averaged along the fin. It is worth mentioning here that Laor and Kalman [13] who presented the fin efficiency for annular fins of different profile subjects to temperature-dependent heat transfer coefficient used the same definition for the fin efficiency. However, Laor and Kalman [13] used two different ways to evaluate the actual and maximum possible heat transfer from the fin. They calculate the actual heat transfer from the fin by applying Fourier's law at the base and utilizing the derivative of the temperature profile at the fin base (i.e., by calculating the heat that enters the fin by conduction at its base). On the other hand, they calculated the maximum possible heat transfer from the fin by applying Newton's law of cooling and considered that the entire surface of the fin was at the same temperature of the base. In this regard, they mentioned that the evaluation of the maximum possible heat transfer from the fin was reduced to calculating the surface area of the fin which implies that they considered the heat transfer coefficient along the fin to be constant when they evaluated the maximum possible heat transfer from the fin. Since the heat transfer coefficient in this study is temperature dependent, it has its maximum value at the fin base. The consideration of maximum possible heat transfer coefficient and temperature along the fin as those at the base would result in a large value of the maximum possible heat transfer from the fin compared to that calculated based on the actual heat transfer coefficient from the fin while considering only the temperature to be the maximum possible of that at the base. The author believed that the first consideration has been adopted by Laor and Kalman [13] in their analysis of fin efficiency for different types of fins subject to temperature heat transfer coefficient and this explains why they obtained lower fin efficiencies compared to those obtained for pertinent cases subject to constant heat transfer coefficient. The second consideration has been adopted in the present work to evaluate the fin efficiency as the ratio of the actual heat transfer to the maximum possible heat transfer from the fin based on the actual heat transfer coefficient and the maximum possible fin surface temperature as that of the fin base. The author of the present work believes that this second consideration rather than the first consideration is closer to the definition of fin efficiency that was presented by Gardner [8] in his pioneer work and followed by all textbooks and researchers including Ullmann and Kalman [9] to whom the present work has been compared.

Results obtained for annular fins subject to variable heat transfer coefficient are presented in Figs. 2–6 for annular fins of rectangular, constant heat flow area, triangular, concave and convex profiles, respectively. For the first two profiles (rectangular, constant heat flow area profiles with radius ratio = 1), the available analytical solution has been plotted as dotted lines, in Figs. 2 and 3, to illustrate the deviation between the fin efficiency based on the constant heat transfer coefficient and that is based on the variable heat transfer coefficient as a function of the local temperature along the fin. The operating parameters investigated for all fin profiles considered in this paper are the radius ratio of the annular fin, r_o/r_b, and the dimensionless parameter m. The ranges of these two parameters ($r_o/r_b = 1$–5 and $m =$ 0–5) considered in this paper are typically the same ranges of both parameters considered by Gardner [8],

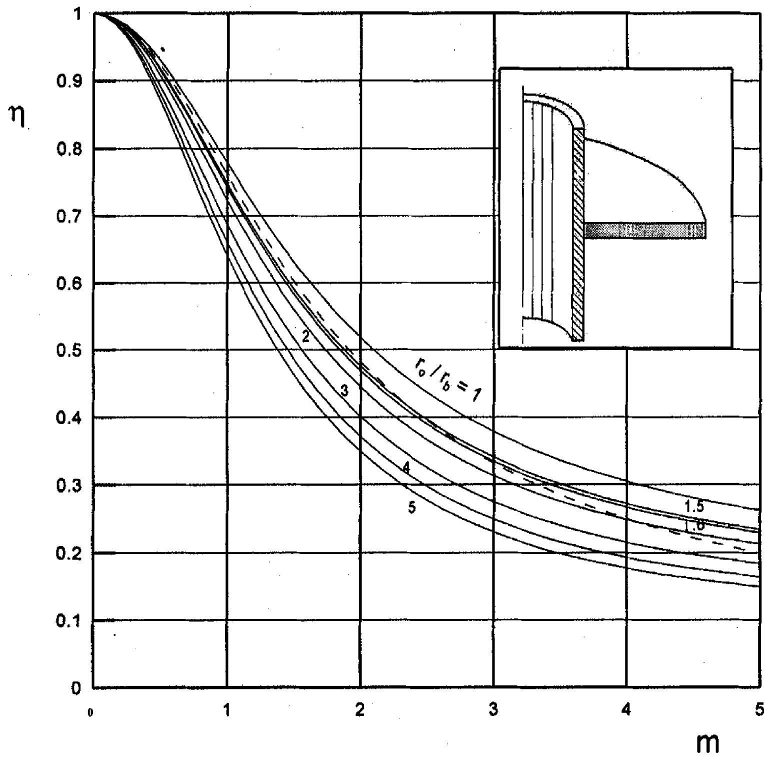

Fig. 2. Fin efficiency with dimensionless parameter m for annular fin with rectangular profile with variable heat transfer coefficient, ------ analytical solution for ratio = 1, constant heat transfer coefficient.

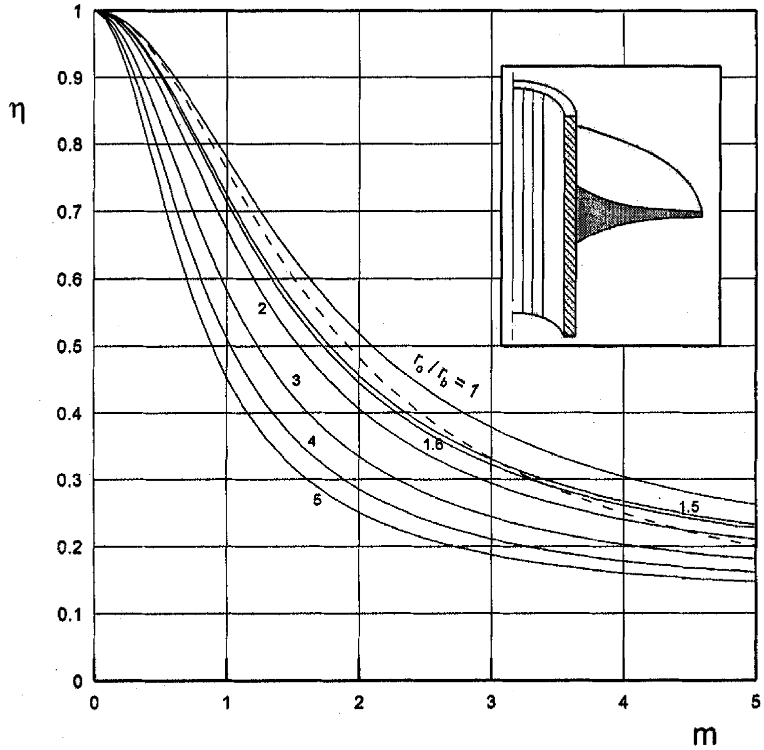

Fig. 3. Fin efficiency with dimensionless parameter m for annular fin with constant heat flow area profile with variable heat transfer coefficient, ------ analytical solution for ratio = 1, constant heat transfer coefficient.

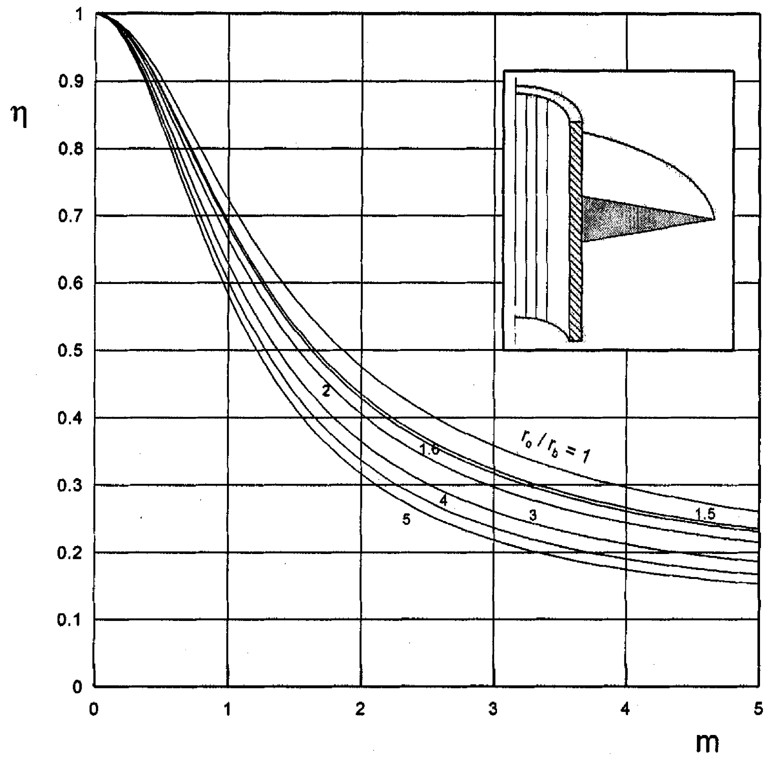

Fig. 4. Fin efficiency with dimensionless parameter m for annular fin with triangular profile with variable heat transfer coefficient.

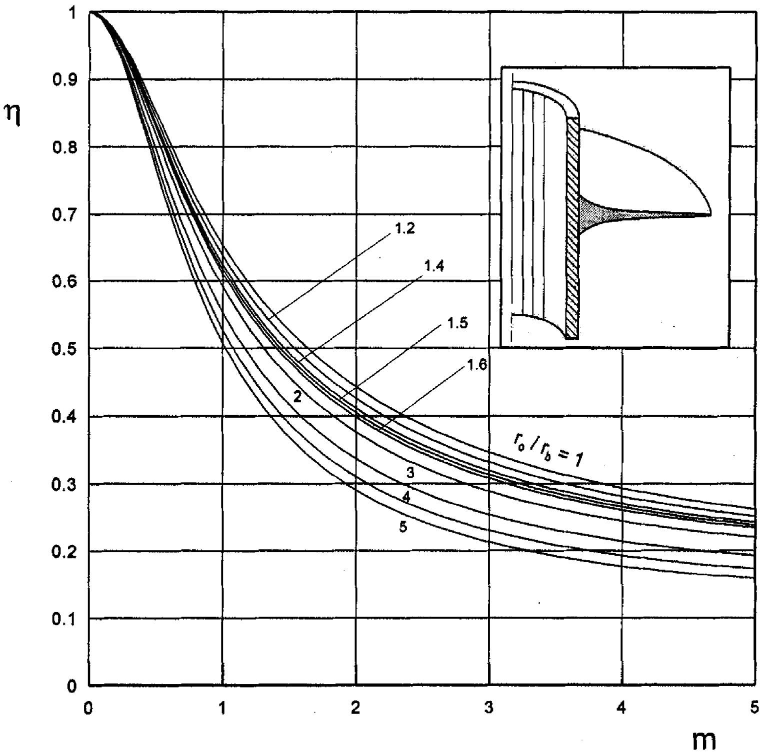

Fig. 5. Fin efficiency with dimensionless parameter m for annular fin with concave parabolic profile with variable heat transfer coefficient.

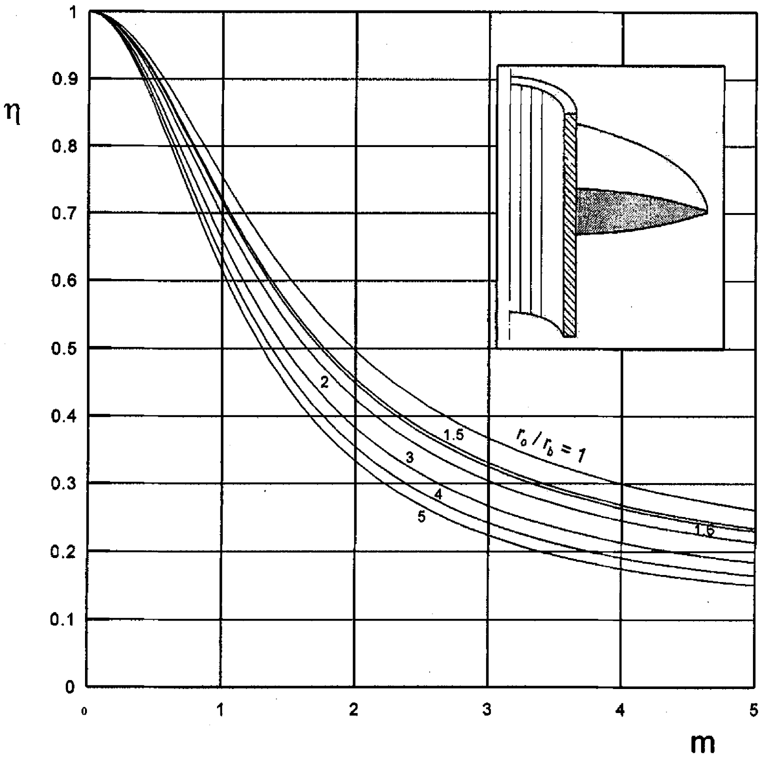

Fig. 6. Fin efficiency with dimensionless parameter m for annular fin with convex profile with variable heat transfer coefficient.

Table 1
Comparison of the fin efficiency for annular fins with rectangular profile

Profile	Gardner [8]	Present	Difference (%)
Radius ratio	$m = 1$		
1	0.7615	0.7792	2.274
1.5	0.7231[a]	0.7484	3.384
2	0.6920	0.7243	4.460
3	0.6420	0.6883	6.731
4	0.6105	0.6622	7.802
5	0.5846[a]	0.6419	8.922
Radius ratio	$m = 2$		
1	0.4820	0.5190	7.130
1.5	0.4308[a]	0.4753	9.365
2	0.3915	0.4452	12.069
3	0.3320	0.4015	17.319
4	0.3115	0.3714	16.119
5	0.2846[a]	0.3495	18.564
Radius ratio	$m = 3$		
1	0.3310	0.3787	12.585
1.5	0.2846[a]	0.3406	16.450
2	0.2560	0.3132	18.263
3	0.2142	0.2751	22.129
4	0.1895	0.2493	23.985
5	0.1769[a]	0.2305	23.258
Radius ratio	$m = 4$		
1	0.2498	0.3050	18.008
1.5	0.2154[a]	0.2722	20.869
2	0.1873	0.2485	24.633
3	0.1560	0.2156	27.631
4	0.1316	0.1934	31.932
5	0.1231[a]	0.1771	30.502
Radius ratio	$m = 5$		
1	0.2000	0.2622	23.719
1.5	0.1693[a]	0.2341	27.698
2	0.1445	0.2133	32.265
3	0.1189	0.1843	35.469
4	0.1000	0.1644	39.167
5	0.0923[a]	0.1499	38.416

[a] Results obtained by Ullmann and Kalman [9].

Table 2
Comparison of the fin efficiency for annular fins with constant area profile

Profile	Gardner [8]	Present	Difference (%)
Radius ratio	$m = 1$		
1	0.7615	0.7792	2.274
1.5	0.7000[a]	0.7256	3.522
2	0.6526	0.6743	3.214
3	0.5684	0.5846	2.766
4	0.5053	0.5124	1.393
5	0.4492[a]	0.4547	1.218
Radius ratio	$m = 2$		
1	0.4820	0.5190	7.130
1.5	0.4092[a]	0.4568	10.408
2	0.3632	0.4070	10.776
3	0.2947	0.3353	12.110
4	0.2474	0.2865	13.646
5	0.2154[a]	0.2513	14.308
Radius ratio	$m = 3$		
1	0.3310	0.3787	12.585
1.5	0.2769[a]	0.3304	16.186
2	0.2421	0.2944	17.752
3	0.1926	0.2445	21.219
4	0.1611	0.2115	23.862
5	0.1385[a]	0.1880	26.331
Radius ratio	$m = 4$		
1	0.2498	0.3050	18.008
1.5	0.2092[a]	0.2675	21.776
2	0.1737	0.2403	27.729
3	0.1400	0.2030	31.020
4	0.1126	0.1782	36.816
5	0.1000[a]	0.1604	37.651
Radius ratio	$m = 5$		
1	0.2000	0.2622	23.719
1.5	0.1646[a]	0.2324	29.168
2	0.1316	0.2108	37.585
3	0.1105	0.1811	38.984
4	0.0895	0.1613	44.528
5	0.0746[a]	0.1469	49.220

[a] Results obtained by Ullmann and Kalman [9].

Ullmann and Kalman [9] and many heat transfer textbooks with the exception that Gardner [8] considered radius ratio range of 1–4 only. It is worth mentioning here that radius ratio of 1 represents cases when the radius of curvature of the annular fin approaches infinity which is practically the straight fin. So, the results for all fin profiles for radius ratio of 1 are practically the results of a straight fin of the pertinent profile. For the two cases of annular fin with rectangular profile and constant heat flow area profile with radius ratio = 1, the results are typically those for a straight fin with rectangular profile. So, the analytical solution for these two cases is the same.

Moreover the fin efficiency calculated using constant heat transfer coefficient along the fin (as given by Gardner [8], Ullmann and Kalman [9] and most of the heat transfer textbooks) has been compared with the efficiency calculated through the present work based on the variable heat transfer coefficient along the fin as function of the temperature, for selected values of the

Table 3
Comparison of the fin efficiency for annular fins with triangular profile

Profile	Ullmann and Kalman [9]	Present	Difference (%)
Radius ratio	$m = 1$		
1	–	0.7236	–
1.5	0.6615	0.6900	4.131
2	0.6230	0.6647	6.274
3	0.5769	0.6279	8.119
4	–	0.6019	–
5	0.5138	0.5823	11.761
Radius ratio	$m = 2$		
1	–	0.4753	–
1.5	0.3769	0.4349	13.336
2	0.3462	0.4055	14.634
3	0.3000	0.3647	17.740
4	–	0.3367	–
5	0.2500	0.3161	20.908
Radius ratio	$m = 3$		
1	–	0.3581	–
1.5	0.2615	0.3227	18.942
2	0.2385	0.2968	19.644
3	0.1923	0.2606	26.213
4	–	0.2362	–
5	0.1538	0.2183	29.550
Radius ratio	$m = 4$		
1	–	0.2965	–
1.5	0.1923	0.2662	27.763
2	0.1615	0.2438	33.743
3	0.1446	0.2121	31.826
4	–	0.1905	–
5	0.1138	0.1746	34.824
Radius ratio	$m = 5$		
1	–	0.2607	–
1.5	0.1615	0.2344	31.074
2	0.1462	0.2146	31.902
3	0.1154	0.1864	38.097
4	–	0.1669	–
5	0.0846	0.1524	44.474

Table 4
Comparison of the fin efficiency for annular fins with concave parabolic profile

Profile	Ullmann and Kalman [9]	Present	Difference (%)
Radius ratio	$m = 1$		
1	–	0.6537	–
1.5	0. 5692	0.6176	7.832
2	0.5431	0.5909	8.096
3	0.4923	0.5530	10.972
4	–	0.5271	–
5	0. 4385	0.5079	13.665
Radius ratio	$m = 2$		
1	–	0.4430	–
1.5	0. 3652	0.4049	10.707
2	0.3154	0.3768	16.298
3	0.2692	0.3373	20.185
4	–	0.3108	–
5	0. 2231	0.2913	23.421
Radius ratio	$m = 3$		
1	–	0.3468	–
1.5	0. 2538	0.3139	19.147
2	0.2154	0.2894	25.587
3	0.1800	0.2545	29.347
4	–	0.2310	–
5	0. 1462	0.2135	31.543
Radius ratio	$m = 4$		
1	–	0.2943	–
1.5	0. 1938	0.2663	27.218
2	0.1662	0.2451	32.210
3	0.1308	0.2146	39.065
4	–	0.1934	–
5	0. 1046	0.1776	41.099
Radius ratio	$m = 5$		
1	–	0.2630	–
1.5	0. 1538	0.2389	35.609
2	0.1308	0.2203	40.635
3	0.1154	0.1930	40.229
4	–	0.1613	–
5	0. 0846	0.1592	46.869

dimensionless parameter m, is summarized in Tables 1–4 for different radius ratios. There were no available results in the literature, up to the best knowledge of the author, for the fin efficiency of an annular fin with convex parabolic profile. So, a comparison of the present work with previous work for this case was not possible and is not provided.

These results show that the assumption of constant heat transfer coefficient along the fin in heat transfer situations that is dominated by natural convection mode would lead to a real underestimation of the fin efficiency. Thus, the use of the fin efficiency predicted by the present study based on variable heat transfer coefficient as a function of the local temperature along the fin would result in a considerable reduction of the fin material since the surface required would be reduced. The results also show that the deviation between the fin efficiency calculated based on constant heat transfer coefficient and that calculated based on variable heat transfer coefficient increases with the increase of the dimensionless parameter m as well as the radius ratio. This deviation reaches, at $m = 5$, a value of 39.2% for annular fin with rectangular profile of radius ratio of 4, 49.2% for annular fin with constant heat flow area profile and radius ratio of 5, 44.5% for annular fin with triangular profile of radius ratio of 5 and 46.9% for annular fin with concave parabolic profile of radius ratio of 5.

4. Conclusion

Heat transfer from annular fins subject to locally variable heat transfer coefficient has been studied. The local heat transfer coefficient as a function of the local temperature has been obtained using the available correlations of natural convection for plates. The results showed that the assumption of constant heat transfer coefficient along the fin in such cases leads to a significant underestimation of the fin efficiency. The deviation between the fin efficiency calculated based on constant heat transfer coefficient and that calculated based on variable heat transfer coefficient increases with both the dimensionless parameter m and the radius ratio of the fin. The use of the present results by the designers of heat transfer equipment that involve annular fins subject to natural convection heat transfer mode would result in a considerable reduction in the extended surface area and hence a significant reduction in the weight and size of the heat transfer equipment.

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